

# Connection between a possible fifth force and the direct detection of Dark Matter

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If there is a fifth force in the dark sector and dark sector particles interact non-gravitationally with ordinary matter, quantum corrections generically lead to a fifth force in the visible sector. We show how the strong experimental limits on fifth forces in the visible sector constrain the direct detection cross section, and the strength of the fifth force in the dark sector. If the latter is comparable to gravity, the spin-independent direct detection cross section must typically be  $\lesssim 10^{-55}$  cm<sup>2</sup>. The anomalous acceleration of ordinary matter falling towards dark matter is also constrained:  $\eta_{\text{OM-DM}} \lesssim 10^{-8}$ .

In recent years, the possibility has arisen that there might be a fifth force acting in the dark sector. (For a theoretical perspective see, e.g., [1, 2] and references therein.) A force in the dark sector arises in many of the extensions of the standard model. It could be short range (as in most dark matter models) or long range (through the existence of a very light boson coupling to dark matter particles); in the latter case it is known as a “fifth force”. Light bosons such as the dilaton appear naturally in supersymmetry and string theory, although if such light bosons exist they are constrained to couple only very weakly to ordinary matter thanks to the Eöt-Wash experiments discussed below.

There are observational motivations to consider a long-range attractive non-gravitational force between dark matter (DM) concentrations. Such a force may resolve some discomforts with conventional  $\Lambda$ CDM. If the range of the fifth force is large compared to the scale of structure formation, it effectively increases the strength of the gravitational interaction for the dark matter by a factor  $\equiv (1+\beta)$ , and thus would accelerate structure formation. This would be helpful because:

- Recent cosmological simulations using a reduced  $\sigma_8$  value compatible with the WMAP year 5 cosmology [3], predict  $z = 0$  halo concentrations distinctly lower than the Millennium Run using the larger WMAP1 value of  $\sigma_8$ , and find “very significant discrepancies with X-ray observations of groups and clusters of galaxies.” Enhancing the effective gravitational attraction with a fifth force would mimic the effect of a larger  $\sigma_8$  at recombination and might eliminate this problem.
- The number of superclusters observed in SDSS data appears to be an order of magnitude larger than predicted by  $\Lambda$ CDM simulations [4]; accelerated structure formation would reduce this discrepancy.
- As noted in [2], a fifth force would tend to clear out the voids; Ref. [5] confirms this in a simulation. This may improve agreement with  $\Lambda$ CDM [6], although the existence of a discrepancy has been challenged [7].
- A variety of observations, for instance the lack of evidence in the Milky Way for a major merger, is hard to rec-

onile with the amount of accretion predicted in  $\Lambda$ CDM. Accelerated structure formation reduces late-time accretion, simply because it leaves less to be accreted later [5].

- The number of satellites in a galaxy such as the Milky Way is predicted to be an order of magnitude larger than is observed. This “substructure problem” is ameliorated by a 5th force, by reducing the stellar content of dwarf galaxies and making them harder to find. This is because baryons – not feeling the 5th force – are relatively less-bound to dark matter concentrations than in conventional theory, reducing the amount of bound gas and lowering the star formation rate in dwarf galaxies, and increasing the tidal loss of the stars that do form. However observational bias may also be responsible for the apparent contradiction [8].

Experimental constraints on the existence of equivalence-principle-violating forces for dark matter are relatively weak [9]. On a sub-galactic scale, Kesden and Kamionkowski pointed out that a difference in acceleration between DM and stars would change the distribution of stars in the tidal tails of the Sagittarius dwarf galaxy and claimed a limit about 10% the strength of gravity [10]. However this claim must be set aside for the time being, because the distances of the stars in the Stream have been found to have significant systematic error [11] and until those distances are revised, it is difficult to draw any conclusions using the Sagittarius Stream. With the original distances there was a difficulty reconciling the observed line-of-sight velocities of the stars in the leading stream, which implied a prolate Dark Matter halo, with the precession of the debris orbital plane, which implied an oblate halo [12]. This discrepancy might be resolved by an additional attractive force, or may disappear when the stellar distances are corrected. On a galaxy-cluster scale, the apparent need for a fifth force to account for the reported velocity [13] of the components of the merging “bullet” cluster, IE0657-56, [14] has been removed by a better determination of the velocity [15]. Work is presently underway to extract a limit on or preferred values for the strength and range of a fifth force from

this system (L. Berezhiani and GRF).

While there is room to consider a fifth force in the dark sector, constraints on new forces in the visible sector are very strict. Precise experiments performed by the Eöt-Wash Group used a torsion balance to study the differential acceleration of a Be-Ti test-body pair in the fields of the Earth, the Sun and our Galaxy [16]. They tested the universality of free fall, leading to a limit on a new Yukawa type force between ordinary matter particles on astronomical length scales

$$\alpha_5 \equiv \frac{g_{\text{eff}}^2}{4\pi u^2 G} \leq 10^{-10} \equiv \alpha_{5,\text{ul}} , \quad (1)$$

where  $u$  is the atomic mass unit and  $g_{\text{eff}}$  is the Yukawa coupling of ordinary matter to a new light boson. The upper limit  $\alpha_{5,\text{ul}}$  depends on the specific coupling of the new force, and the quoted value corresponds to the least stringent limit; more typical values are two orders of magnitude smaller [17]. Such a strong limit arises naturally if some symmetry forbids a tree-level coupling between ordinary matter and the light boson which mediates the fifth force, and we assume here that this is the case. If the tree-level coupling does not vanish, then our arguments below become even stronger unless there is extreme fine-tuning.

In this paper we show that thanks to the bound (1), a connection exists between the strength of astrophysical effects of a fifth force and laboratory experiments to detect dark matter. Namely, the existence of a fifth force between dark matter particles could set a strong upper bound on the possible direct detection cross section and vice versa. From Figs. 1(a) and 2 it is clear how a fifth force can arise for ordinary matter as a quantum effect involving a fifth force between dark matter particles and the direct detection vertex. The constraint (1) on the effective Yukawa interaction between ordinary matter particles can therefore impose a limit on the product of the DM direct detection cross section and the DM fifth force, which we determine below.

We assume the fifth force boson to be a light scalar particle, because a pseudoscalar particle would give a spin-dependent, and thus non-macroscopic, force, and a vector or axial-vector carrier would give rise to a repulsive force. If dark matter is fermionic and there is a non-gravitational attractive fifth force between dark matter particles, it is most simply modeled as a long-range Yukawa force and we adopt that description here. Reality may be more complex. For instance in chameleon models [18] the new interaction is damped by sufficiently strong concentrations of matter, so in principle it could have an impact on structure formation but not be evident on sub-galactic scales. The bounds derived here would need to be revisited in such models.

Consider a Yukawa interaction between a spin-1/2 dark matter field  $\chi$  and a light scalar  $\phi$  (spin-0 dark matter is

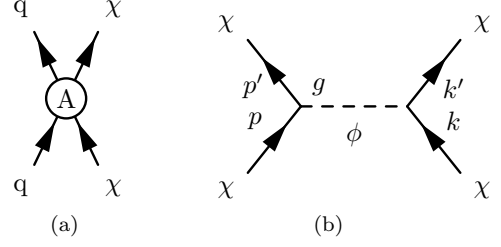


FIG. 1: (a) Direct detection four-fermion vertex coupling quarks and dark matter fermions  $\chi$ . (b) Fifth force Yukawa interaction for fermionic dark matter.

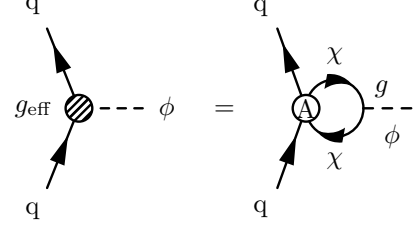


FIG. 2: An effective Yukawa interaction of ordinary matter particles arises due to a quantum correction.

discussed below)

$$\mathcal{L}_{\text{int}} = -g(\phi - \phi_*)\bar{\chi}\chi . \quad (2)$$

Here,  $g$  is a dimensionless constant and  $g\phi_*$  is the mass of a dark matter particle in the absence of a vacuum expectation value  $\langle\phi\rangle$  of the scalar field.

The potential corresponding to this interaction is found by comparing the amplitude of the process in Fig. 1(b) to the Born approximation formula from non-relativistic quantum mechanics: the scattering amplitude in the low-energy approximation, normalized according to non-relativistic conventions is given by

$$\mathcal{M} = \frac{g^2}{|\mathbf{q}|^2 + m_\phi^2} \quad (\mathbf{q} = \mathbf{p}' - \mathbf{p}) , \quad (3)$$

which is simply the Fourier transform of the potential, giving the familiar Yukawa potential (see, e.g., [19])

$$V(r) = -\frac{g^2}{4\pi} \frac{1}{r} e^{-m_\phi r} , \quad (4)$$

in which  $m_\phi$  is the mass of the scalar. At distances small compared to the range of the force,  $m_\phi^{-1}$  in natural units, the force is just proportional to the gravitational attraction, with proportionality constant  $\beta$ :

$$\beta \equiv \frac{g^2/4\pi}{Gm_\chi^2} = \frac{1}{4\pi G(\langle\phi\rangle - \phi_*)^2} = \frac{m_{\text{Pl}}^2}{4\pi(\langle\phi\rangle - \phi_*)^2} , \quad (5)$$

since the mass of the dark matter particle is  $m_\chi = g(\langle\phi\rangle - \phi_*)$ . Notice how  $g$  drops out of equation (5),

leaving the relative strength of the fifth force compared to gravity and the mass of the dark matter particles as two independent quantities. This relationship, derived here from the single particle perspective, was found earlier from density perturbation considerations [2].

A few observations: a) In order for the fifth force to be of the order of the gravitational force, i.e. to be astrophysically relevant,  $\langle\phi\rangle - \phi_*$  should be of the order of the Planck mass. This choice is quite natural, since there is no immediate relation between the scalar and any of the lower mass scales. Since  $\langle\phi\rangle - \phi_*$  appears in the denominator, a smaller value would mean a much larger fifth force. b) With  $\langle\phi\rangle - \phi_*$  of the order of the Planck scale, the dark matter would be very heavy in most scenarios. To get a mass in the electroweak region would require  $g, \tilde{g} \sim O(m_W/m_{\text{Pl}})$ , which presumably should be enforced by some symmetry. For  $g, \tilde{g}$  closer to unity, we can infer that dark matter must be a nonthermal relic. Nonthermal relics are produced in sufficient abundance if their mass is of order the inflaton mass (presumably around  $10^{13}$  GeV) [20]. This would connect fifth force physics with inflation physics, which would be interesting, but not within the scope of this work.

In direct detection experiments, evidence for dark matter is sought in the observation of ordinary matter particles recoiling due to their being scattered by dark matter particles. The most general non-derivative four-fermion operator is given by

$$\mathcal{M} = \sum_i A_i (\bar{q} O_i q) (\bar{\chi} O_i \chi), \quad (6)$$

where the sum is over scalar (S), pseudoscalar (P), vector (V), axial-vector (A), and tensor vertices (T), and the  $A_i$  are constants with dimensions of energy<sup>-2</sup>. If the DM particle is Majorana, the vector and tensor terms are absent in (6); this is commonplace in supersymmetric theories. In the limit of small momenta, the only contributing terms are the scalar, vector, axial-vector, and tensor interactions. These last two correspond to the spin-dependent cross section, which the reasoning below will fail to limit.

The spin-independent (SI) cross section for (fermionic) DM - quark scattering is

$$\sigma_{q\chi}^{\text{SI}} = (A_S + A_V)^2 \frac{m_q^2}{\pi}, \quad (7)$$

in the limit that the DM mass is much larger than  $m_q$  (the effective quark mass,  $\sim \Lambda_{\text{QCD}}$ ).

Figure 2 shows how an effective Yukawa-type fifth force interaction of quarks arises as a consequence of a quantum correction. The correction is made up of two parts: first, the direct detection vertex couples the two quarks to two virtual dark matter particles; second, these dark matter particles couple to the fifth force scalar. For scalar  $\phi$  the scalar and tensor contributions to the sum (6) give

non-vanishing effective couplings

$$g_{\text{eff}} \approx A_{S,T} g \Lambda^2, \quad (8)$$

where  $\Lambda$  is the ultraviolet cut-off, since this diagram would be quadratically divergent if the 4-fermion vertex were pointlike. In reality, we expect the short distance theory to be renormalizable, giving the effective 4-fermi vertex a form-factor. This cuts off the divergence at some model-dependent scale leading to an expression like (7) with  $\Lambda$  for instance of the order of the TeV-scale, the dark matter-scale, or the Planck-scale.

Through an insertion of the Higgs vacuum expectation value into the loop diagram, the V contribution to the sum (6) gives a non-vanishing effective vertex

$$g_{\text{eff}} \approx A_V g m_q m_\chi \ln \Lambda, \quad (9)$$

since this diagram is now superficially logarithmically divergent. The quantum correction in Fig. 2 from the axial-vector term in (6) vanishes. Therefore,  $A_A$  is not constrained and no relation between a fifth force for DM and a fifth force for ordinary matter ensues.

Through (5) we re-express the dark matter Yukawa-coupling as  $g^2 = 4\pi\beta G m_\chi^2$ , where  $\beta$ , the relative strength of the fifth force with respect to gravity, is  $\lesssim O(1)$  from astrophysics [9]. Imposing the experimental limit (1) on the effective vertex (8), leads to

$$A_{S,T}^2 \lesssim \frac{\alpha_{5,\text{ul}}}{\beta} \frac{u^2}{m_\chi^2 \Lambda^4}, \quad (10)$$

or for the vector coupling

$$A_V^2 \lesssim \frac{\alpha_{5,\text{ul}}}{\beta} \frac{u^2}{m_\chi^4 m_q^2 \ln^2 \Lambda}. \quad (11)$$

Since the bounds on  $A_i$  become ever more stringent for higher DM mass scales, we use the smallest  $m_\chi$ , i.e., weak scale DM, to obtain the most conservative limits. If both  $A_S$  or  $A_T$  and  $A_V$  are present in (7), the constraint (11) is dominant and from (7) and (11) we find an upper bound on the product of  $\beta$  and the spin-independent direct detection cross section:

$$\beta \left( \frac{\sigma_{q\chi}^{\text{SI}}}{10^{-43} \text{ cm}^2} \right) \lesssim 10^{-3} \times \left( \frac{\alpha_{5,\text{ul}}}{10^{-10}} \right) \left( \frac{100 \text{ GeV}}{m_\chi} \right)^4 \left( \frac{1}{\ln \Lambda} \right)^2. \quad (12)$$

However, if only the scalar term in (6) is present in (7) – e.g., because DM is Majorana – then (10) leads to the more stringent bound:

$$\beta \left( \frac{\sigma_{q\chi}^{\text{SI}}}{10^{-43} \text{ cm}^2} \right) \lesssim 10^{-12} \times \left( \frac{\alpha_{5,\text{ul}}}{10^{-10}} \right) \left( \frac{m_q}{\Lambda_{\text{QCD}}} \right)^2 \left( \frac{100 \text{ GeV}}{m_\chi} \right)^2 \left( \frac{1 \text{ TeV}}{\Lambda} \right)^4. \quad (13)$$

The product of the spin-dependent (SD) cross section and  $\beta$  is similarly constrained in the case that only a tensor term contributes to the matrix element (6). The SD cross section is like in (7) but with  $(A_S + A_V)^2$  replaced by  $6A_T^2$ . The bound on the product of the SD cross section and  $\beta$  is then of the same form as equation (13), but with the coefficient  $10^{-11}$ .

In the case of spin-0 dark matter, we take the interaction Lagrangian to be  $\mathcal{L}_{\text{int}} = -\tilde{g}m_\chi(\phi - \phi_*)\phi_\chi^2$ , in which  $\phi_\chi$  is the spin-0 dark matter field and the coupling  $\tilde{g}$  is chosen so that the amplitude of the process in Fig. 1(b) is again given by (3), and consequently the potential is the same as for fermionic DM, with  $g \rightarrow \tilde{g}$ . The spin-0 DM mass is  $m_\chi = 2\tilde{g}(\langle\phi\rangle - \phi_*)$ , so the relative strength of the fifth force with respect to gravity at distances  $\ll m_\phi^{-1}$  in the spin-0 case is then  $\tilde{\beta} = \frac{m_{\text{Pl}}^2}{16\pi(\langle\phi\rangle - \phi_*)^2}$ . We parametrize the direct detection vertex as  $\mathcal{M} = \tilde{A}_S m_\chi \bar{q} q \phi_\chi^2$ , leading to the DM-quark cross section  $\sigma_{q\chi}^{\text{SI}} = \tilde{A}_S^2 m_q^2 / 4\pi$ . The diagram in Figure 2 now is superficially logarithmically divergent, which leads us to write  $g_{\text{eff}} \approx \tilde{A}_S \tilde{g} m_\chi^2 \ln \Lambda$ . As before we get an upper bound on the four-fermion vertex:  $\tilde{A}_S^2 \lesssim \frac{\alpha_{5,\text{ul}}}{\tilde{\beta}} \frac{u^2}{m_\chi^6 \ln^2 \Lambda}$ , which leads to

$$\beta \left( \frac{\sigma_{q\chi}^{\text{SI}}}{10^{-43} \text{ cm}^2} \right) \lesssim 10^{-9} \times \left( \frac{\alpha_{5,\text{ul}}}{10^{-10}} \right) \left( \frac{m_q}{\Lambda_{\text{QCD}}} \right)^2 \times \left( \frac{100 \text{ GeV}}{m_\chi} \right)^6 \left( \frac{1}{\ln \Lambda} \right)^2, \quad (14)$$

for scalar DM.

If there is an astrophysically relevant  $\beta$ , these bounds on the spin-independent direct detection cross section are smaller than current experimental limits, which restrict the SI scattering cross section of DM on ordinary matter to be less than  $10^{-43} \text{ cm}^2$  for  $m_\chi = 100 \text{ GeV}$  and less than  $10^{-30} \text{ cm}^2$  for superheavy DM,  $m_\chi = 10^{13} \text{ GeV}$  [21].

Before closing, we note that the effective vertex in Fig. 2 discussed above also leads to a fifth force between ordinary matter (OM) and dark matter proportional to  $gg_{\text{eff}}$ . This implies that the fractional anomalous acceleration of two materials falling towards DM is given by

$$\eta_{\text{OM-DM}} \lesssim 10^{-8} \times \left( \frac{\eta_{\text{OM-OM}} \times \Delta(q/\mu) \times \beta}{10^{-13} \times 10^{-3} \times 1} \right)^{1/2}, \quad (15)$$

where  $\eta_{\text{OM-OM}}$  is the fractional anomalous acceleration for the two materials falling towards OM and  $\Delta(q/\mu)$  is the difference of the ratio of the fifth-force-charge to the mass of the different materials, with the average  $q/\mu$  of the sun equal to one. This limit is more stringent than current experimental bounds,  $\eta_{\text{OM-DM}} \lesssim 10^{-5}$  [16].

To summarize, we have shown here that if there is a cosmologically important dark matter fifth force, then the experimental limit on a fifth force for ordinary mat-

ter essentially excludes the direct detection of dark matter particles in the near future, except through the spin-dependent cross section. Generically, for weak scale dark matter and a gravitational-strength fifth force,  $\sigma_{q\chi}^{\text{SI}} \leq 10^{-55} \text{ cm}^2$ ; in the least constrained case the cross section must be  $\leq 10^{-46} \text{ cm}^2$ . For heavier dark matter, as in nonthermal relics scenarios, the bound is even smaller. Conversely, if the spin-independent interaction of dark matter is observed in laboratory experiments, our argument shows that any fifth force in the dark sector must be too weak to be astrophysically relevant. The tensor part of the spin-dependent cross section is similarly constrained. We have also given a model independent upper bound on a non-gravitational interaction between ordinary matter and dark matter, which is significantly better than current direct experimental limits.

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